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A REPAIRABLE SYSTEM WITH COMMON CAUSE FAILURES AND HUMAN ERRORS

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Abstract: **A Repairable system working in a fluctuating environment is considered. The transition rates are arbitrary. The mathematically formulated problem is solved using the direct integration method for solving partial differential equations. Expression for the reliability function and mean time to system failure are given. Four special cases are discussed.**

Keywords: **Repairable System.**

1. INTRODUCTION

Many authors have discussed a number of systems under stable environmental conditions. In practice, we come across situations where operating systems are subject to failure due to changes in environmental conditions. Due to this, the system may not fail completely but may work in a reduced efficiency state. From this reduced working state, due to further deterioration of environmental conditions, the system may go directly to the failed state or via a reduced working state. If there is a repair facility, the system may be repaired partially or completely to go back to the operating state. Examples of outdoor power systems and communication systems have bee discussed. In Ref (4)

In Ref (4) Three models (reparable and non-reparable) with constant failure and repair rates are considered and the analysis is carried out using Laplace transforms. This paper discusses a more general case of a reparable system with arbitrary repair and failure rates. It is now difficult to solve the problem using Laplace transforms. The problem is solved using Lagrange's method for partial differential equations. Expressions for the reliability function and mean time to system failure (MTSF) are given for several cases.

2. ASSUMPTIONS AND NOTATIONS

(a) The system is working in a fluctuation environment resulting in four States : operating in normal state, operating in, abnormal states with reduced efficiency, failed.

(b) Failure, repair and environmental change over rates are arbitrary.

(c) A repaired unit is as good as new.

O — The system is operating in the normal environmental state.

①, ② — The system is operating in abnormal states. Environmental change over may take place from state 1 to 2 or 2 to 1.

[3] — The failed state.

α_i (y) — transition rate from normal working state "0" to abnormal state 1 : the transition occurs within the time interval (y,y+ dy), i= 1,2

μ_i (x) transition rate from failed state to normal and abnormal states; the transition occurs within the time interval (x,x+dx), i= 1,2

λ_i (y) the transition rate from working state to failed state 3, i= 0,1,2

β_i (x) the transition rate from abnormal state 1 to normal state, i= 1,2

α_3 (y) the transition rate from state 1 to state 2

β_3 (y) the transition rate from state 2 to state 1

P_0 (t) the probability that at time t the system is operating in the normal state

P_i (x,y,t) the probability that the system is in state, "i" at time *t* ; the transitions occur within the time intervals (y,y+dy) and (x,x+dx), 1,2,3

Limits of integration are from 0 to ∞

Following the above assumptions and notation, we obtain the transition diagram shown in fig 1.

3. MATHEMATICAL ANALYSIS

Probability consideration gives the following integro-differential equations (7) associated with the transition diagram

$$\left(\frac{d}{dt} + S_0(y)\right) P_0(t) = S(t) \tag{1}$$

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial t} + S_i(x,y)\right) P_i(x,y,t) = S_{i+3}(x,y,t), \quad i = 1,2 \tag{2}$$

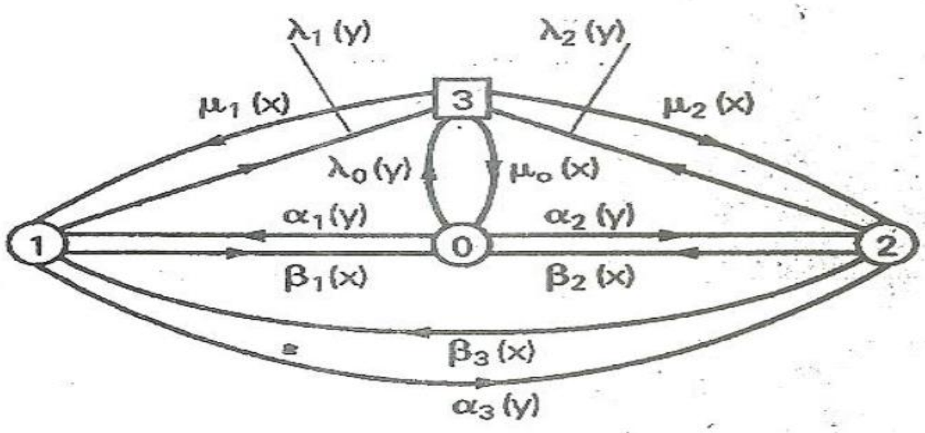

Fig. 1 Transition Diagram

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial t} + S_3(x)\right) P_3(x,y,t) = S_6(x,y,t) \tag{3}$$

With initial and boundary conditions

$$\left.\begin{aligned} &P_0(0) = 1, \text{ otherwise zero} \\ &P_i\,(x,y,0) = 0,\ i = 1,2,3 \end{aligned}\right\} \tag{4}$$

$$P_i\,(0,y,t) = \alpha_i\,(y)\,P_0(t) + K\int \alpha_{i+1}(y)\,P_{i-1}\,(x,y,t)\,dx,\ i=1,2 \tag{5}$$

$$P_3(0,y,t) = \gamma_0\,(y)\,P_0\,(t) + \int\left[\sum_{1}^{2} \lambda_i(y)\,P_i\,(x,y,t)\ dx,\right] \tag{6}$$

Where

$$K = \begin{cases} 0 & \text{for } i=1 \\ 1 & \text{for } i=2 \end{cases}$$

$$S_0\,(y) = \lambda_0\,(y) + \sum_{i=1}^{2} \alpha_i\,(y)$$

$$S\,(t) = \int\left(\sum_{1}^{2}\beta_i(x)\,P_i(x,y,t)\right) dx\,dy + \int \mu_0\,(x)\,P_3\,(x,y,t)\,dx\,dy$$

$$S_1\,(y) = \alpha_3\,(y) + \beta_1\,(x) + \lambda_1\,(y)$$

$$S_2(x,y) = \sum_{2}^{3}\beta_i(x) + \lambda_2\,(y)$$

$$S_3(x) = \sum_{0}^{2} \mu_i\,(x)$$

$$S_4(x,y,t) = \alpha_1\,(y)\,P_0\,(t) + \beta_3\,(x)\,P_2\,(x,y,t) + \mu_1\,(x)\,P_3\,(x,y,t)$$

$$S_5(x,y,t) = \alpha_2\,(y)\,P_0\,(t) + \alpha_3\,(y)\,P_1\,(x,y,t) + \mu_2\,(x)\,P_3\,(x,y,t)$$

$$S_6(x,y,t) = \lambda_0\,(y)\,P_0\,(t) + \sum_{1}^{2} \lambda_i\,(y)\,P_i\,(x,y,t)$$

Using (5) and (6), Lagrange's method for partial differential equations (8) yields the following solution for equations (2) and (3):

$$P_i(x,y,t) = \left[\int S_i+3(x,y,t) \exp \left(\int S_i(.)dx \right) dx + \phi_i(t-x) \right] \exp \left[-\int S_i(.)\,dx), \right] i = 1,2,3 \qquad (7)$$

Where $\phi_i(t) = P_i(0,x,t)$ and $S_i(.)$ is replaced accordingly.

The solution of equation (1) [using (4)] is given by

$$P_o(t) = \left[1+ \int S(t) \exp(S_o t)\,dt \right] \exp(-S_o t). \qquad (8)$$

In the above analysis P_3 (.) is obtained in terms of P_0 (.), P_1 (.) and P_2 (.) while P_2 (.) is given in terms of P_0 (.) and P_1 (.) Thus P_3 (.) is obtained in terms of P_0 (.) and P_1 (.) Substituting the values of P_2 (.) and P_3 (.) in terms of P_0 (.) and P_1 (.), equation (7) gives P_1 (.) in terms of P_0 (.). All the probabilities P_i (.) (i=1,2,3) are thus obtained in terms of P_0 (.) which is given by the integral equation (8).

The reliability function R (.) and mean time to system failure (MTSF) are given by

$$R(t) = P_o(t) + \int \left[\sum_{1}^{2} P_i(x,y,t) \right] dx\,dy$$

$$MTSF = \int R(t)\,dt.$$

IV. SPECIAL CASES

Case I : Steady State

When t - ∞, d/dt and ∂/∂t-0. Equations (1)-(3) reduce to
$S_0(y)\,P_0=S$ (9)

$$\left(\frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} + S_i(x,y) \right) P_i(x,y) = S_i+3(x,y), \quad i = 1,2 \qquad (10)$$

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + S_3(x) \right) P_3(x,y) = S_6(x,y), \qquad (11)$$

Where S, S_4(.), S_5(.), S_6(.) are independent of t.

Equations (10) and (11) have the following solution:

$$P_i\,(x,y) = \left[\int S_{i+3}\,(.)\,\exp\left[\int S_i(.)\,dx\right) dx \right.$$
$$\left. +\varphi_i\,(y\text{-}x) \right] \exp\left(- \int S_i(.)\,dx\right),\ i= 1,2,3 \qquad (12)$$

Where ϕ_i (y) = P_i (0,y)

Arguing as in Section 3, all the probabilities Pi (.), i =1,2,3, are obtained in terms of P_0 which can be obtained using the normalizing

condition, i.e. the sum of all the probabilities is equal to one ($\sum_0^3 Pi\,(.) = 1$)

The availability function A, is given by

$$A_v = P_o + \int \left(\sum_1^2 P_i\,(x,y)\,dx\right) dy$$

Case II

If the transition rates are constant, then replacing the transition rate parameters by constant parameters and solving (after taking Laplace Transforms) we get the result reported in Ref. (4). In this case, the problem can easily be solved using the foregoing method of Section 3.

Case III

When the transition rates are constant, the steady state probabilities are given by (putting d/dt= 0 and solving the equations:

$P_i = P_0\, D_i/D$, i = 1,2,3

Where ,

$D = S_2S_3S_4\text{-}\ \lambda_2\mu_2S_2\text{-}\beta_3\alpha_3S_4\text{-}\beta_3\lambda_1\mu_2$
$\quad\text{-}\mu_1\alpha_3\lambda_2+\ \lambda_1S_3\mu_1$

$D_1= \alpha_3S_3S_4\text{-}\alpha_3\lambda_2\mu_2+\beta_3\alpha_3S_4+\lambda_0\mu_2\beta_3$
$\quad+\ \mu_1\alpha_2\lambda_2+\ \lambda_0\mu_1S_3$

$D_2= \alpha_2S_2S_4 +\ \lambda_0\mu_2S_4+\alpha^2S_4+\alpha_3\lambda_1\mu_2$
$\quad+\ \mu_1\alpha_3\lambda_0\text{-}\lambda_1\alpha_2\mu_1$

$D_3= \lambda_0S_2S_3+\lambda_2\alpha_2S_2\text{-}\beta_3\alpha_3\lambda_0+\ \lambda_1\alpha_2\beta_3$
$\quad+\ \alpha^2\,\lambda_2+\ \lambda_1S_2S_3$

All S are independent of x, y and t.

Using the normalizing condition ($\Sigma^3{}_0\,P_i$=1), we get

$$P_o{=}D \Big/ \left[D+\sum_1^3 D_i \right]$$

$$A_v{=} \left[1+\sum_1^2 Di/D \right] \Big/ \left[1+\sum_1^3 D_i/D \right]$$

Case IV

If the system is non-reparable from the failed state, the various probabilities, reliability function availability and MTSF are obtained by putting $\mu_1(x) = 0$, i=1,2 in the foregoing results.

The analysis provides straightforward results for the system with arbitrary transition rates. The results are useful for reliability engineers. Since the inversion of the probability Laplace transform is difficult, Lagrange's integration method is more suitable for the analysis of complex systems. Moreover, since there are advanced techniques available for solving integral equations, the method is suitable for computing the results. The analysis may be extended to systems working under a multiple environment.

REFERENCES

[1] E. Balagurusamy, Reliability Engineering. Tata Mc-Graw Hill, New Delhi, India (1984).

[2] B.S. Dhillon and C. Singh, Engineering Reliability-New Techniques and Applications. John Wiley, New York (1981).

[3] B.S Dhillon and J. Natesan, Stochastic analysis of outdoor power systems in fluctuation environment. Microelectron, Reliab.23(5), 867-881(1983).

[4] N. U Prabhu, Queus and Inventories- A Study of Their Basic Stochastic Processes. John Wiley, New York (1965).

[5] I.N. Sneddon, Elements of Partial Differntial Equations McGraw-Hill, New York (1957).

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